1. Bewegingsvergelijkingen/Wetten van Newton/Cirkelbeweging

P5.21 (a) Isolate either mass

$$T + mg = ma = 0$$
$$|T| = |mg|.$$

The scale reads the tension T,

so

$$T = mg = 5.00\text{ kg}\left(9.80\text{ m/s}^2\right) = \boxed{49.0\text{ N}}.$$

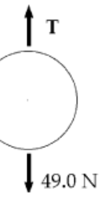

FIG. P5.21(a)

(b) Isolate the pulley

$$\mathbf{T}_2 + 2\mathbf{T}_1 = 0$$
$$T_2 = 2|T_1| = 2mg = \boxed{98.0\text{ N}}.$$

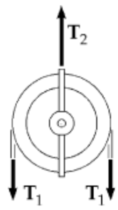

FIG. P5.21(b)

(c) $\sum \mathbf{F} = \mathbf{n} + \mathbf{T} + m\mathbf{g} = 0$

Take the component along the incline

$$\mathbf{n}_x + \mathbf{T}_x + m\mathbf{g}_x = 0$$

or

$$0 + T - mg\sin 30.0° = 0$$
$$T = mg\sin 30.0° = \frac{mg}{2} = \frac{5.00(9.80)}{2}$$
$$= \boxed{24.5\text{ N}}.$$

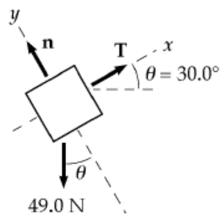

FIG. P5.21(c)

P5.46 (Case 1, impending upward motion)
Setting

$$\sum F_x = 0: \quad P\cos 50.0° - n = 0$$
$$f_{s,\,\max} = \mu_s n: \quad f_{s,\,\max} = \mu_s P\cos 50.0°$$
$$= 0.250(0.643)P = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° - 0.161P - 3.00(9.80) = 0$$
$$P_{\max} = \boxed{48.6\text{ N}}$$

(Case 2, impending downward motion)
As in Case 1,

$$f_{s,\,\max} = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° + 0.161P - 3.00(9.80) = 0$$
$$P_{\min} = \boxed{31.7\text{ N}}$$

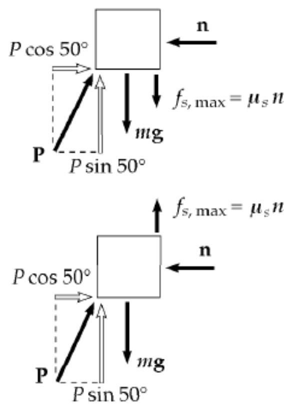

FIG. P5.46

P6.60 For the block to remain stationary, $\sum F_y = 0$ and $\sum F_x = ma_r$.

$$n_1 = (m_p + m_b)g \text{ so } f \le \mu_{s1} n_1 = \mu_{s1}(m_p + m_b)g.$$

At the point of slipping, the required centripetal force equals the maximum friction force:

$$\therefore (m_p + m_b)\frac{v_{\max}^2}{r} = \mu_{s1}(m_p + m_b)g$$

or $v_{\max} = \sqrt{\mu_{s1} r g} = \sqrt{(0.750)(0.120)(9.80)} = 0.939 \text{ m/s}$.

For the penny to remain stationary on the block:

$$\sum F_y = 0 \Rightarrow n_2 - m_p g = 0 \text{ or } n_2 = m_p g$$

and $\sum F_x = ma_r \Rightarrow f_p = m_p \dfrac{v^2}{r}$.

When the penny is about to slip on the block, $f_p = f_{p,\max} = \mu_{s2} n_2$

or $\mu_{s2} m_p g = m_p \dfrac{v_{\max}^2}{r}$

$$v_{\max} = \sqrt{\mu_{s2} r g} = \sqrt{(0.520)(0.120)(9.80)} = 0.782 \text{ m/s}$$

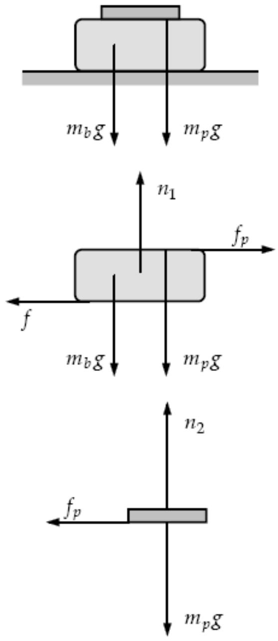

FIG. P6.60

This is less than the maximum speed for the block, so the penny slips before the block starts to slip. The maximum rotation frequency is

$$\text{Max rpm} = \frac{v_{\max}}{2\pi r} = (0.782 \text{ m/s})\left[\frac{1 \text{ rev}}{2\pi(0.120 \text{ m})}\right]\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{62.2 \text{ rev/min}}.$$

2. Behoud van impuls, energie en impulsmoment

P9.60 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or $(3.00 \text{ kg})v_{\text{wedge}} + (0.500 \text{ kg})(+4.00 \text{ m/s}) = 0$

so $v_{\text{wedge}} = \boxed{-0.667 \text{ m/s}}$

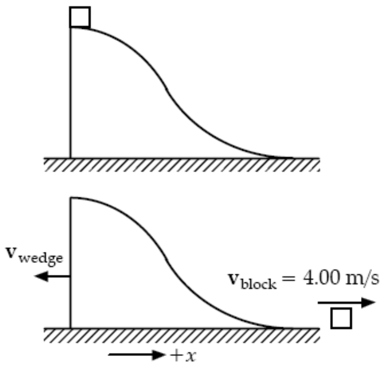

FIG. P9.60

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

or $\left[0 + m_1 g h\right] + 0 = \left[\frac{1}{2} m_1 (4.00)^2 + 0\right] + \frac{1}{2} m_2 (-0.667)^2$ which gives $\boxed{h = 0.952 \text{ m}}$.

P8.10 Choose the zero point of gravitational potential energy of the object-spring-Earth system as the configuration in which the object comes to rest. Then because the incline is frictionless, we have $E_B = E_A$:

$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

or $$0 + mg(d + x)\sin\theta + 0 = 0 + 0 + \frac{1}{2}kx^2.$$

Solving for d gives $$d = \boxed{\frac{kx^2}{2mg\sin\theta} - x}.$$

P8.36 $\sum F_y = n - mg\cos 37.0° = 0$

$\therefore n = mg\cos 37.0° = 400\text{ N}$

$f = \mu n = 0.250(400\text{ N}) = 100\text{ N}$

$-f\Delta x = \Delta E_{\text{mech}}$

$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$

$\Delta U_A = m_A g\left(h_f - h_i\right) = (50.0)(9.80)(20.0\sin 37.0°) = 5.90\times 10^3$

$\Delta U_B = m_B g\left(h_f - h_i\right) = (100)(9.80)(-20.0) = -1.96\times 10^4$

$\Delta K_A = \frac{1}{2}m_A\left(v_f^2 - v_i^2\right)$

$\Delta K_B = \frac{1}{2}m_B\left(v_f^2 - v_i^2\right) = \frac{m_B}{m_A}\Delta K_A = 2\Delta K_A$

Adding and solving, $\Delta K_A = \boxed{3.92\text{ kJ}}$.

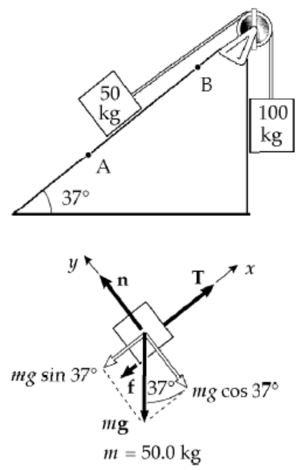

FIG. P8.36

P11.35 (a) $L_i = mv\ell \qquad \sum\tau_{\text{ext}} = 0$, so $L_f = L_i = \boxed{mv\ell}$

$L_f = (m + M)v_f\ell$

$v_f = \left(\frac{m}{m + M}\right)v$

(b) $K_i = \frac{1}{2}mv^2$

$K_f = \frac{1}{2}(M + m)v_f^2$

$v_f = \left(\frac{m}{M + m}\right)v \Rightarrow$ velocity of the bullet and block

Fraction of K lost $= \dfrac{\frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2}{M+m}v^2}{\frac{1}{2}mv^2} = \boxed{\frac{M}{M + m}}$

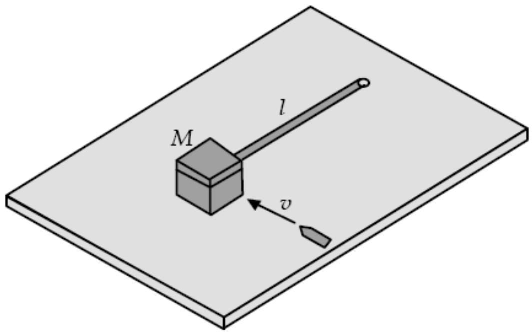

FIG. P11.35

3. Wet van Coulomb/Het elektrische veld

P23.6 We find the equal-magnitude charges on both spheres:

$$F = k_e \frac{q_1 q_2}{r^2} = k_e \frac{q^2}{r^2} \qquad \text{so} \qquad q = r\sqrt{\frac{F}{k_e}} = (1.00\ \text{m})\sqrt{\frac{1.00\times10^4\ \text{N}}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 1.05\times10^{-3}\ \text{C}\,.$$

The number of electron transferred is then

$$N_{\text{xfer}} = \frac{1.05\times10^{-3}\ \text{C}}{1.60\times10^{-19}\ \text{C}/e^-} = 6.59\times10^{15}\ \text{electrons}\,.$$

The whole number of electrons in each sphere is

$$N_{\text{tot}} = \left(\frac{10.0\ \text{g}}{107.87\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right)\left(47\ e^-/\text{atom}\right) = 2.62\times10^{24}\ e^-\,.$$

The fraction transferred is then

$$f = \frac{N_{\text{xfer}}}{N_{\text{tot}}} = \left(\frac{6.59\times10^{15}}{2.62\times10^{24}}\right) = \boxed{2.51\times10^{-9}} = 2.51\ \text{charges in every billion.}$$

P23.55 (a) Let us sum force components to find

$$\sum F_x = qE_x - T\sin\theta = 0\,,\ \text{and}\ \sum F_y = qE_y + T\cos\theta - mg = 0\,.$$

Combining these two equations, we get

$$q = \frac{mg}{\left(E_x\cot\theta + E_y\right)} = \frac{\left(1.00\times10^{-3}\right)(9.80)}{(3.00\cot 37.0°+5.00)\times10^5} = 1.09\times10^{-8}\ \text{C}$$

$$= \boxed{10.9\ \text{nC}}$$

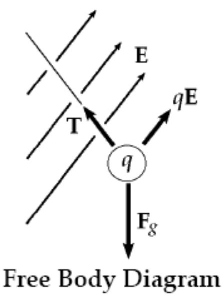

FIG. P23.55

(b) From the two equations for $\sum F_x$ and $\sum F_y$ we also find

$$T = \frac{qEx}{\sin 37.0°} = 5.44\times10^{-3}\ \text{N} = \boxed{5.44\ \text{mN}}\,.$$

P23.72

$$d\mathbf{E} = \frac{k_e dq}{x^2 + (0.150\ \text{m})^2}\left(\frac{-x\hat{\mathbf{i}} + 0.150\ \text{m}\hat{\mathbf{j}}}{\sqrt{x^2 + (0.150\ \text{m})^2}}\right) = \frac{k_e\lambda\left(-x\hat{\mathbf{i}} + 0.150\ \text{m}\hat{\mathbf{j}}\right)dx}{\left[x^2 + (0.150\ \text{m})^2\right]^{3/2}}$$

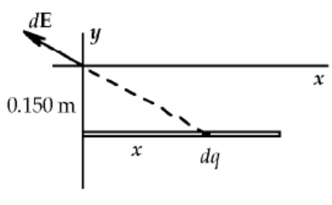

FIG. P23.72

$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = k_e\lambda\int_{x=0}^{0.400\ \text{m}} \frac{\left(-x\hat{\mathbf{i}} + 0.150\ \text{m}\hat{\mathbf{j}}\right)dx}{\left[x^2 + (0.150\ \text{m})^2\right]^{3/2}}$$

$$\mathbf{E} = k_e\lambda\left[\left.\frac{+\hat{\mathbf{i}}}{\sqrt{x^2 + (0.150\ \text{m})^2}}\right|_0^{0.400\ \text{m}} + \left.\frac{(0.150\ \text{m})\hat{\mathbf{j}}x}{(0.150\ \text{m})^2\sqrt{x^2 + (0.150\ \text{m})^2}}\right|_0^{0.400\ \text{m}}\right]$$

$$\mathbf{E} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(35.0\times10^{-9}\ \text{C/m}\right)\left[\hat{\mathbf{i}}(2.34-6.67)\ \text{m}^{-1} + \hat{\mathbf{j}}(6.24-0)\ \text{m}^{-1}\right]$$

$$\mathbf{E} = \left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)\times10^3\ \text{N/C} = \boxed{\left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)\ \text{kN/C}}$$

4. Wet van Gauss/Het elektrisch potentiaal

P24.19 If $R \le d$, the sphere encloses no charge and $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \boxed{0}$.

If $R > d$, the length of line falling within the sphere is $2\sqrt{R^2 - d^2}$

so $\Phi_E = \boxed{\frac{2\lambda\sqrt{R^2-d^2}}{\epsilon_0}}$.

P24.57 (a) $\oint \mathbf{E}\cdot d\mathbf{A} = E\left(4\pi r^2\right) = \frac{q_{\text{in}}}{\epsilon_0}$

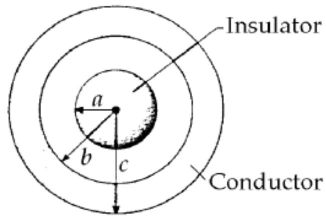

FIG. P24.57

For $r < a$, $q_{\text{in}} = \rho\left(\frac{4}{3}\pi r^3\right)$

so $E = \boxed{\frac{\rho r}{3\epsilon_0}}$.

For $a < r < b$ and $c < r$, $q_{\text{in}} = Q$.

So $E = \boxed{\frac{Q}{4\pi r^2 \epsilon_0}}$.

For $b \le r \le c$, $E = 0$, since $\boxed{E = 0}$ inside a conductor.

(b) Let $q_1 =$ induced charge on the inner surface of the hollow sphere. Since $E = 0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b \le r \le c$ must be zero.

Therefore, $q_1 + Q = 0$ and $\sigma_1 = \frac{q_1}{4\pi b^2} = \boxed{\frac{-Q}{4\pi b^2}}$.

Let $q_2 =$ induced charge on the outside surface of the hollow sphere. Since the hollow sphere is uncharged, we require

$$q_1 + q_2 = 0 \quad \text{and} \quad \sigma_2 = \frac{q_1}{4\pi c^2} = \boxed{\frac{Q}{4\pi c^2}}.$$

5. Condensatoren

P26.21 (a) $$\frac{1}{C_s}=\frac{1}{15.0}+\frac{1}{3.00}$$

$C_s = 2.50\ \mu\text{F}$

$C_p = 2.50+6.00 = 8.50\ \mu\text{F}$

$$C_{eq}=\left(\frac{1}{8.50\ \mu\text{F}}+\frac{1}{20.0\ \mu\text{F}}\right)^{-1}=\boxed{5.96\ \mu\text{F}}$$

(b) $Q = C\Delta V = (5.96\ \mu\text{F})(15.0\ \text{V}) = \boxed{89.5\ \mu\text{C}}$ on $20.0\ \mu\text{F}$

$$\Delta V=\frac{Q}{C}=\frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}}=4.47\ \text{V}$$

$15.0-4.47=10.53\ \text{V}$

$Q = C\Delta V = (6.00\ \mu\text{F})(10.53\ \text{V}) = \boxed{63.2\ \mu\text{C}}$ on $6.00\ \mu\text{F}$

$89.5-63.2=\boxed{26.3\ \mu\text{C}}$ on $15.0\ \mu\text{F}$ and $3.00\ \mu\text{F}$

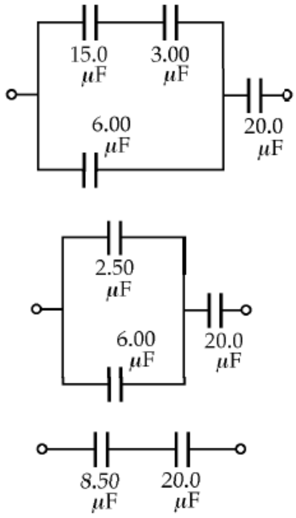

FIG. P26.21

P26.58 (a) We use Equation 26.11 to find the potential energy of the capacitor. As we will see, the potential difference ΔV changes as the dielectric is withdrawn. The initial and final energies are $U_i=\frac{1}{2}\left(\frac{Q^2}{C_i}\right)$ and $U_f=\frac{1}{2}\left(\frac{Q^2}{C_f}\right)$.

But the initial capacitance (with the dielectric) is $C_i=\kappa C_f$. Therefore, $U_f=\frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right)$.

Since the work done by the external force in removing the dielectric equals the change in potential energy, we have $W=U_f-U_i=\frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right)-\frac{1}{2}\left(\frac{Q^2}{C_i}\right)=\frac{1}{2}\left(\frac{Q^2}{C_i}\right)(\kappa-1)$.

To express this relation in terms of potential difference ΔV_i, we substitute $Q=C_i(\Delta V_i)$, and evaluate: $W=\frac{1}{2}C_i(\Delta V_i)^2(\kappa-1)=\frac{1}{2}(2.00\times10^{-9}\ \text{F})(100\ \text{V})^2(5.00-1.00)=\boxed{4.00\times10^{-5}\ \text{J}}$.

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f=\frac{Q}{C_f}$.

Substituting $C_f=\frac{C_i}{\kappa}$ and $Q=C_i(\Delta V_i)$ gives $\Delta V_f=\kappa\Delta V_i=5.00(100\ \text{V})=\boxed{500\ \text{V}}$.

Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

P26.72 Assume a potential difference across a and b, and notice that the potential difference across the $8.00\ \mu\text{F}$ capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the following circuit:

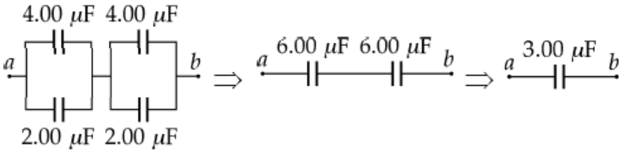

FIG. P26.72

$C_{ab} = \boxed{3.00\ \mu\text{F}}$.

6. Elektrische stroom

P27.56 We find the drift velocity from $I = nqv_d A = nqv_d \pi r^2$

$$v_d = \frac{I}{nq\pi r^2} = \frac{1\,000\ \text{A}}{8.49\times10^{28}\ \text{m}^{-3}\left(1.60\times10^{-19}\ \text{C}\right)\pi\left(10^{-2}\ \text{m}\right)^2} = 2.34\times10^{-4}\ \text{m/s}$$

$$v = \frac{x}{t} \qquad t = \frac{x}{v} = \frac{200\times10^3\ \text{m}}{2.34\times10^{-4}\ \text{m/s}} = 8.54\times10^8\ \text{s} = \boxed{27.0\ \text{yr}}$$

P28.59 Let the two resistances be x and y.

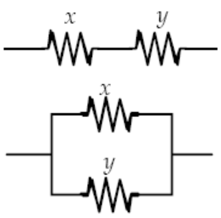

FIG. P28.59

Then, $R_s = x + y = \frac{\mathscr{P}_s}{I^2} = \frac{225\ \text{W}}{(5.00\ \text{A})^2} = 9.00\ \Omega$ $\qquad y = 9.00\ \Omega - x$

and $R_p = \frac{xy}{x+y} = \frac{\mathscr{P}_p}{I^2} = \frac{50.0\ \text{W}}{(5.00\ \text{A})^2} = 2.00\ \Omega$

so $\frac{x(9.00\ \Omega - x)}{x + (9.00\ \Omega - x)} = 2.00\ \Omega$ $\qquad x^2 - 9.00x + 18.0 = 0$.

Factoring the second equation, $\qquad (x - 6.00)(x - 3.00) = 0$

so $\qquad x = 6.00\ \Omega$ or $x = 3.00\ \Omega$.

Then, $y = 9.00\ \Omega - x$ gives $\qquad y = 3.00\ \Omega$ or $y = 6.00\ \Omega$.

The two resistances are found to be $\boxed{6.00\ \Omega}$ and $\boxed{3.00\ \Omega}$.

P28.21 We name currents I_1, I_2, and I_3 as shown.

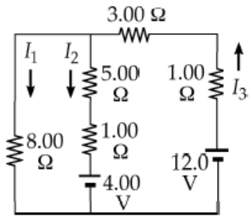

FIG. P28.21

From Kirchhoff's current rule, $I_3 = I_1 + I_2$.

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$12.0\text{ V} - (4.00)I_3 - (6.00)I_2 - 4.00\text{ V} = 0$

$8.00 = (4.00)I_3 + (6.00)I_2$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$$-(6.00)I_2 - 4.00\text{ V} + (8.00)I_1 = 0 \qquad (8.00)I_1 = 4.00 + (6.00)I_2.$$

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = 1.33I_1 - 0.667 \end{cases}$$

and to the single equation $8 = 4I_1 + 13.3I_1 - 6.67$

$$I_1 = \frac{14.7\text{ V}}{17.3\ \Omega} = 0.846\text{ A}. \qquad \text{Then} \qquad I_2 = 1.33(0.846\text{ A}) - 0.667$$

and $\quad I_3 = I_1 + I_2 \quad$ give $\quad \boxed{I_1 = 846\text{ mA},\ I_2 = 462\text{ mA},\ I_3 = 1.31\text{ A}}$.

All currents are in the directions indicated by the arrows in the circuit diagram.

P28.71 (a) After steady-state conditions have been reached, there is no DC current through the capacitor.

Thus, for R_3: $\quad \boxed{I_{R_3} = 0\ (\text{steady-state})}$.

For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12-kΩ and 15-kΩ resistors in series:

For R_1 and R_2: $\quad I_{(R_1+R_2)} = \dfrac{\varepsilon}{R_1 + R_2} = \dfrac{9.00\text{ V}}{(12.0\text{ k}\Omega + 15.0\text{ k}\Omega)} = \boxed{333\ \mu\text{A}\ (\text{steady-state})}$.

(b) After the transient currents have ceased, the potential difference across C is the same as the potential difference across $R_2 (= IR_2)$ because there is no voltage drop across R_3. Therefore, the charge Q on C is

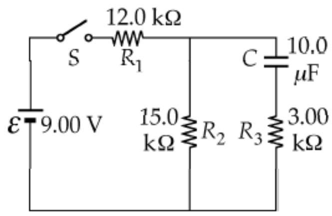

FIG. P28.71(b)

$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10.0\ \mu\text{F})(333\ \mu\text{A})(15.0\text{ k}\Omega)$$
$$= \boxed{50.0\ \mu\text{C}}.$$

continued on next page

(c) When the switch is opened, the branch containing R_1 is no longer part of the circuit. The capacitor discharges through (R_2+R_3) with a time constant of $(R_2+R_3)C=(15.0\text{ k}\Omega+3.00\text{ k}\Omega)(10.0\ \mu\text{F})=0.180\text{ s}$. The initial current I_i in this discharge circuit is determined by the initial potential difference across the capacitor applied to (R_2+R_3) in series:

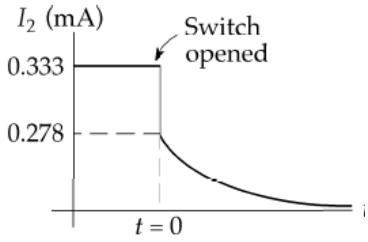

FIG. P28.71(c)

$$I_i=\frac{(\Delta V)_C}{(R_2+R_3)}=\frac{IR_2}{(R_2+R_3)}=\frac{(333\ \mu\text{A})(15.0\text{ k}\Omega)}{(15.0\text{ k}\Omega+3.00\text{ k}\Omega)}=278\ \mu\text{A}.$$

Thus, when the switch is opened, the current through R_2 changes instantaneously from 333 μA (downward) to 278 μA (downward) as shown in the graph. Thereafter, it decays according to

$$I_{R_2}=I_i e^{-t/(R_2+R_3)C}=\boxed{(278\ \mu\text{A})e^{-t/(0.180\text{ s})}\ (\text{for } t>0)}.$$

(d) The charge q on the capacitor decays from Q_i to $\frac{Q_i}{5}$ according to

$$q=Q_i e^{-t/(R_2+R_3)C}$$
$$\frac{Q_i}{5}=Q_i e^{(-t/0.180\text{ s})}$$
$$5=e^{t/0.180\text{ s}}$$
$$\ln 5=\frac{t}{180\text{ ms}}$$
$$t=(0.180\text{ s})(\ln 5)=\boxed{290\text{ ms}}$$

7. Magnetisme

P29.64 Call the length of the rod L and the tension in each wire alone $\frac{T}{2}$. Then, at equilibrium:

$$\sum F_x=T\sin\theta-ILB\sin 90.0°=0 \quad\text{or}\quad T\sin\theta=ILB$$
$$\sum F_y=T\cos\theta-mg=0, \quad\text{or}\quad T\cos\theta=mg$$
$$\tan\theta=\frac{ILB}{mg}=\frac{IB}{(m/L)g} \quad\text{or}\quad B=\frac{(m/L)g}{I}\tan\theta=\boxed{\frac{\lambda g}{I}\tan\theta}$$

P29.17 The magnetic force on each bit of ring is $Id\mathbf{s} \times \mathbf{B} = IdsB$ radially inward and upward, at angle θ above the radial line. The radially inward components tend to squeeze the ring but all cancel out as forces. The upward components $IdsB\sin\theta$ all add to $\boxed{I2\pi rB\sin\theta \text{ up}}$.

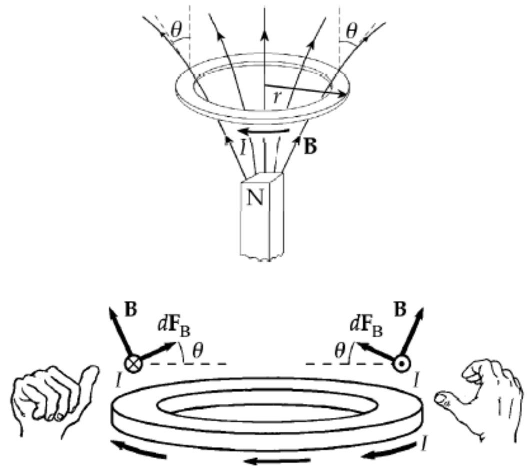

FIG. P29.17

P30.21 Each wire is distant from P by

$(0.200 \text{ m})\cos 45.0° = 0.141 \text{ m}$.

Each wire produces a field at P of equal magnitude:

$$B_A = \frac{\mu_0 I}{2\pi a} = \frac{(2.00 \times 10^{-7} \text{ T}\cdot\text{m/A})(5.00 \text{ A})}{(0.141 \text{ m})} = 7.07 \ \mu\text{T}.$$

Carrying currents into the page, A produces at P a field of 7.07 μT to the left and down at –135°, while B creates a field to the right and down at – 45°. Carrying currents toward you, C produces a field downward and to the right at – 45°, while D's contribution is downward and to the left. The total field is then

$4(7.07 \ \mu\text{T})\sin 45.0° = \boxed{20.0 \ \mu\text{T}}$ toward the bottom of the page

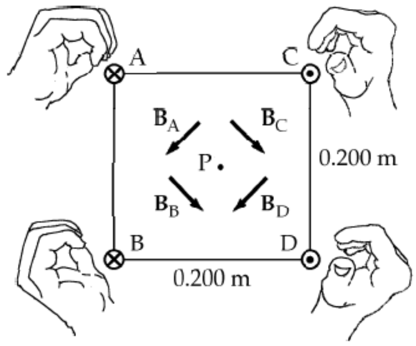

FIG. P30.21

P30.67 By symmetry of the arrangement, the magnitude of the net magnetic field at point P is $B = 8B_{0x}$ where B_0 is the contribution to the field due to current in an edge length equal to $\frac{L}{2}$. In order to calculate B_0, we use the Biot-Savart law and consider the plane of the square to be the yz-plane with point P on the x-axis. The contribution to the magnetic field at point P due to a current element of length dz and located a distance z along the axis is given by Equation 30.3.

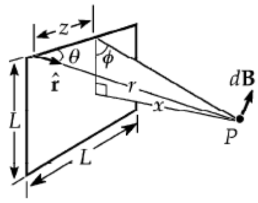

FIG. P30.67

$$\mathbf{B}_0 = \frac{\mu_0 I}{4\pi} \int \frac{d\boldsymbol{\ell} \times \hat{\mathbf{r}}}{r^2}.$$

From the figure we see that

$$r = \sqrt{x^2 + \left(L^2/4\right) + z^2} \text{ and } \left|d\boldsymbol{\ell} \times \hat{\mathbf{r}}\right| = dz \sin\theta = dz\sqrt{\frac{\left(L^2/4\right) + x^2}{\left(L^2/4\right) + x^2 + z^2}}.$$

By symmetry all components of the field $\mathbf{B}$ at P cancel except the components along x (perpendicular to the plane of the square); and

$$B_{0x} = B_0 \cos\phi \text{ where } \cos\phi = \frac{L/2}{\sqrt{\left(L^2/4\right) + x^2}}.$$

$$\text{Therefore, } \mathbf{B}_{0x} = \frac{\mu_0 I}{4\pi} \int_0^{L/2} \frac{\sin\theta \cos\phi dz}{r^2} \text{ and } B = 8B_{0x}.$$

Using the expressions given above for $\sin\theta\cos\phi$, and r, we find

$$B = \frac{\mu_0 I L^2}{2\pi\left(x^2 + \left(L^2/4\right)\right)\sqrt{x^2 + \left(L^2/2\right)}}.$$