

Problem 1 (oral)

Let $X = l^1(\mathbb{N})$ and let $\chi_n = \mathbb{1}_{\{1,2,\dots,n\}}$. Note that $X = (l^\infty(\mathbb{N}))^*$. Does χ_n converge in the weak $*$ topology? What is its limit? Does it converge in the weak topology?

Problem 2

Let $H = L^2[0, 1]$ and T an integral operator with kernel $K(x, y) := x + y$. Is T self adjoint? Is it compact? What is its spectrum? What are the eigenvectors to its nonzero eigenvalues? Compute the dimension of the kernel.

Problem 3

Let $H = l^2(\mathbb{N})$ and let $V_n x(k) = x(n + k)$. Show it converges to zero in the strong operator topology but V_n^* does not converge in the strong operator topology.

Show that if W_n are isometries and $W_n \rightarrow W$ in the strong operator topology, W is an isometry.

Problem 4

Let $X = C^1[0, 1], Y = C[0, 1]$ equipped with the supremum norm. Let $T : X \rightarrow Y : f \rightarrow f'$. Show that the graph of T is closed yet T is not continuous.