# Managerial economics
# Exercises

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## 1 Consumer choice

### 1.1 Problem

Utility function of consumer: $U(X,Y) = AX^{\alpha}Y^{\beta}$.
($A, \alpha, \beta$ parameters, $X$ = amount of goods $x$, $Y$ = amount of goods $y$)
Budget constraint of consumer: $P_X X + P_Y Y = I$.
($I$ = income, $P_i$ is price of good $i$)

1. Write down her constrained optimization problem.
2. Derive the first order condition(s).
3. Solve for optimal consumption.
4. How does consumption of $Y$ change if $P_X$ changes?

### 1.2 Solution

**1.**

$$Y = \frac{I - P_X X}{P_Y} \Rightarrow U(X) = AX^{\alpha}\left(\frac{I - P_X X}{PY}\right)^{\beta}$$

**2.**

$$d = AX^{\alpha}Y^{\beta} + \lambda\left(I - P_X X - P_Y Y\right)$$

First order conditions:

1. $\frac{\partial d}{\partial X} = \alpha AX^{\alpha-1}Y^{\beta} - \lambda P_X = 0$
2. $\frac{\partial d}{\partial Y} = \beta AX^{\alpha}Y^{\beta-1} - \lambda P_Y = 0$
3. $\frac{\partial d}{\partial \lambda} = I - P_X X - P_Y Y = 0$

**3.** Extract $\lambda$ from second conditon:

$$\beta AX^{\alpha}Y^{\beta-1} - \lambda P_Y = 0 \Rightarrow \lambda = \frac{\beta AX^{\alpha}Y^{\beta-1}}{P_Y}$$

Extract $Y$ from third condition:

$$I - P_X X - P_Y Y = 0 \Rightarrow Y = \frac{I - P_X X}{P_Y}$$

Fill in $\lambda$ and $Y$ in first condition:

$$\begin{aligned}
& \alpha A X^{\alpha-1} Y^{\beta} - \lambda P_X = 0 \\
\Rightarrow \quad & \alpha A X^{\alpha-1} Y^{\beta} - \frac{\beta A X^{\alpha} Y^{\beta-1}}{P_Y} P_X = 0 \\
\Rightarrow \quad & \alpha Y = \beta X \frac{P_X}{P_Y} \\
\Rightarrow \quad & \alpha \frac{I - P_X X}{P_Y} = \beta X \frac{P_X}{P_Y} \\
\Rightarrow \quad & \alpha I = (\alpha + \beta) X P_X
\end{aligned}$$

Optimal consumption:

$$X = \frac{\alpha I}{(\alpha + \beta) P_X}$$

Same reasoning for $Y$ gives:

$$Y = \frac{\beta I}{(\alpha + \beta) P_Y}$$

**4.** Consumption of $Y$ doesn't change.

# 2 Firm supply decision

## 2.1 Problem

You can sell as much as you want at the competitive price $P$.
Cost of producing amount $Q$: $TC(Q) = F + \omega Q + \delta Q^2$.
$(F \geq 0, \omega > 0, \delta > 0)$

1. What is your average cost of production if you produce $\hat{Q}$ units?
2. What is your marginal cost of production if you produce $\hat{Q}$ units?
3. What are your profits $\pi$ at an arbitrary level of production $Q$?
4. At what level of production are average costs minimized?
5. Under what conditions would you produce in the long run?
6. Under what conditions would you produce in the short run?

## 2.2 Solution

**1.**

$$AC(\hat{Q}) = \frac{TC(\hat{Q})}{\hat{Q}} = \frac{F}{\hat{Q}} + \omega + \delta\hat{Q}$$

**2.**

$$MC(\hat{Q}) = \frac{d}{dQ}TC(\hat{Q}) = \omega + 2\delta\hat{Q}$$

**3.**

$$\pi = PQ - (F + \omega Q + \delta Q^2)$$

**4.** $AC(Q)$ is minimized if $\frac{d}{dQ}AC(Q) = 0$.

$$\Rightarrow \quad \frac{-F}{Q^2} + \delta = 0$$
$$\Rightarrow \quad Q = \sqrt{\frac{F}{\delta}}$$

**5.**

$$P \geq AVC(Q) \Rightarrow P \geq \omega + \delta\sqrt{\frac{F}{\delta}} = \omega + \sqrt{\delta F}$$

**6.**

$$P \geq AC(Q) \Rightarrow P \geq F\sqrt{\frac{\delta}{F}} + \omega + \delta\sqrt{\frac{F}{\delta}} = 2\sqrt{\delta F} + \omega$$

# 3 Cost accounting

## 3.1 Problem

Acounting statement:

| | A | B | Total |
|---|---|---|---|
| Revenue | 1000 | 2000 | 3000 |
| Variable cost | 200 | 1500 | 1700 |
| Gross profit | 800 | 500 | 1300 |
| Fixed cost | | | 1000 |
| Profit | | | 300 |

To find out what is profitable and what is not, you need to assign fixed costs to your products.

1. by using the share of sales as the criterion
2. by using the share of Gross profits as the criterion

Which product line(s) should be (dis)continued using either of the criteria?

## 3.2 Solution

**1.**

| | A | B |
|---|---|---|
| $FC$ | 333.33 | 666.67 |
| Profit | 466.67 | -166.67 |

Line A should be continued. Line B should be discontinued.
**BUT** Next year, B is closed, and $FC = 1000$. Then, $\pi_A = 1000 - 200 - 1000 = -200$. Thus, both lines should be continued.

**2.**

| | A | B |
|---|---|---|
| $FC$ | 615.38 | 384.62 |
| Profit | 184.62 | 115.38 |

Lines A and B should be continued.

# 4 Monopoly pricing

## 4.1 Problem

Inverse demand: $P = 13 - Q$ ($P$ = price, $Q$ = quantity)
Constant marginal cost: $MC = 1$
Fixed cost of production: $F$

1. What is your profit function?
2. What is your first order condition?
3. What is your optimal quantity?
4. What is your optimal price and profit?
5. Under what conditions do you want to be in business?
6. How would you answers change if the inverse demand was $P = 18 - Q$?

## 4.2 Solution

**1.**

$$\pi(Q) = Q(13 - Q) - (Q + F) = -Q^2 + 12Q - F$$

**2.**

$$\frac{\partial \pi}{\partial Q} = -2Q + 12 = 0$$

**3.**

$$Q^* = 6$$

**4.**

$$P^* = 13 - Q^* = 7$$

$$\pi(Q^*) = -36 + 72 - F = 36 - F$$

**5.**

$$F \leq 36$$

**6.**

$$\pi(Q) = -Q^2 + 17Q - F$$

$$\frac{\partial \pi}{\partial Q} = -2Q + 17 = 0$$

$$Q^* = 8.5, P^* = 9.5, \pi(Q^*) = -72.25 + 144.5 - F = 72.25 - F$$

$$F \leq 72.25$$

# 5 Bundling

## 5.1 Problem

Demand side:

| Segment | # households | Educ. channel | Music channel | Bundle |
|---|---|---|---|---|
| Conservative | 4000 | \$20 | \$2 | \$22 |
| Mainstream | 6000 | \$11 | \$11 | \$22 |

Constant marginal cost: $MC$

Maximize profits using

1. pure bundling (only sell bundle of two channels together)
2. separate prices (only sell each channel separately)
3. mixed bundling (combination of pure bundling and seperate priced)

Under what circumstances will you choose which pricing scheme, and what will your profits be?

## 5.2 Solution

**1.** The only price that you want to consider is 22, and hence the profits are

$$\pi^{PB} = 10000(22 - 2MC) = 220000 - 20000MC.$$

**2a.** The prices you want to consider are 2 and 11, and hence the profits are

$$\pi_M^{SP}(2) = 10000(2 - MC) = 20000 - 10000MC,$$

$$\pi_M^{QP}(11) = 6000(11 - MC) = 66000 - 6000MC.$$

As the revenues are larger, it is clear that the price for the Music channel is 11.

**2b.** The prices you want to consider are 11 and 20, and hence the profits are

$$\pi_E^{SP}(11) = 10000(11 - MC) = 110000 - 10000MC,$$

$$\pi_E^{SP}(20) = 4000(20 - MC) = 80000 - 4000MC.$$

You want to set price 11 if

$$\pi_E^{SP}(11) \geq \pi_E^{SP}(20),$$

$$\text{or } 110000 - 10000MC \geq 80000 - 4000MC$$

$$\text{or } 5 \geq MC.$$

Otherwise, you want to set price 20.

**3.** It is a safe bet that you want to sell only the Educational channel to the Conservatives, and both channels to the Mainstream households.
The highest price you can ask for the Educational channes is 20, and 22 for the bundle. Therefore, your profits would be

$$\pi^{MB} = 4000(20 - MC) + 6000(22 - 2MC) = 212000 - 16000MC.$$

You don't want to sell the Music channel separately, so you price it at 12.

Benefits for Conservatives:

| Educ. channel | Music channel | Bundle |
|---|---|---|
| **20-20 = 0** | 2-12 = -10 | 22-22=0 |

Benefits for Mainstream:

| Educ. channel | Music channel | Bundle |
|---|---|---|
| 11-20 = -9 | 11-12 = -1 | **22-22=0** |

**answer.** If $MC > 20$, you can't make profits. So you sell nothing.

If $MC > 11$, you want to choose separate prices and only sell to Conservatives.

Note that $\pi^{MB} > \pi^{SP}$ if $MC \leq 5$. Thus mixed bundling dominates separate prices.
If $MC \leq 2$, pure bundling gives higher profits than mixed bundling. If $MC > 2$, the opposite is true.

If $5 < MC \leq 11$, we need to compare mixed bundling profits to profits from separate prices. This yields

$$\pi^{MB} - \pi^{SP} = 212000 - 16000MC - (146000 - 10000MC).$$

This is positive if and only if $MC < 11$.

| | $MC \leq 2$<br>PB | $2 < MC \leq 5$<br>MB | $5 < MC \leq 11$<br>MB | $11 < MC \leq 20$<br>SP | $MC > 20$<br>not selling |
|---|---|---|---|---|---|
| Price bundle | **22** | **22** | **22** | 23 | 23 |
| Price Music channel | 12 | 12 | 12 | 12 | 12 |
| Price Educ. channel | 21 | **20** | **20** | **20** | 21 |

# 6 Cournot competition

## 6.1 Problem

Inverse demand function: $P = 13 - Q$. ($P$ = price, $Q$ = quantity)
Constant marginal cost: $MC = 1$.
Two-stage game:

1. You have to pay $F > 0$ to enter the industry.
2. You face competitors and you all decide quantities (as in Cournot competition).

How large can $F$ be if you expect to face $N$ competitors in the second stage of the game?

## 6.2 Solution

**Profit**

$$\pi_i = PQ_i - Q_i = (P-1)Q_i = (12 - (Q_1 + \ldots + Q_{N+1}))Q_i$$

**First order condition**

$$\begin{aligned} & 12 - (Q_1 + \ldots + Q_{i-1} + Q_{i+1} + \ldots + Q_{N+1}) - 2Q_i = 0 \\ \Rightarrow \quad & Q_i = 6 - \frac{1}{2}(Q_1 + \ldots + Q_{i-1} + Q_{i+1} + \ldots + Q_{N+1}) \end{aligned}$$

**Calculating quantity**

$$\begin{aligned} & Q_1^* = \ldots = Q_i^* = \ldots = Q_{N+1}^* \\ \Rightarrow \quad & Q_i^* = 6 - \frac{1}{2}(NQ_i^*) \\ \Rightarrow \quad & \left(1 + \frac{N}{2}\right) Q_i^* = 6 \\ \Rightarrow \quad & \frac{N+2}{N} Q_i^* = 6 \\ \Rightarrow \quad & Q_i^* = \frac{12}{N+2} \end{aligned}$$

**Calculating profit**

$$\pi_i = \left(12 - (N+1)\frac{12}{N+2}\right)\frac{12}{N+2} = \frac{144}{(N+2)^2}$$

**Answer**

$$F \leq \frac{144}{(N+2)^2}$$