Question 1

[1] Write the simplest Feynman diagram of the interaction between a photon and a electron, and the matrix element that corresponds to this interaction.

[2] Consider the dynamics (4-momenta), and prove that there is no scattering if the photon and the electron are on-shell (you can choose a convenient Lorentz-frame if that is helpful).

[3] Allow then momenta of the photon that do not satisfy the mass-shell condition (off-shell photons), and replace its polarization vector ε^μ by p^μ. Show that the matrix element then vanishes.

[4] Consider the electroweak interactions, and the analogous interaction of the off-shell vector bosons with fermions. Do you also get zero in this case when the polarization vector of the vector boson is replaced by p^μ? What is the essential property of the fermions involved in this process that determines this result?

[5] The vanishing of the amplitudes with gauge bosons whose polarization vector is replaced by the momentum is a property of interactions of massless gauge fields. Does this fit with what you know about electroweak interactions?

[6] How does this go for strong interactions? Why is this consistent with what you know about the particles that interact and the properties of the gluons?

Question 2

Consider the non-Abelian gauge theory SU(2) of the electroweak interactions.

[1] Write the covariant derivative $\mathcal{D}_\mu$ acting on a doublet fermion.

[2] Which physical particles consitute such fermion doublet in the standard model?

[3] Calculate explicitly the commutator of two covariant derivatives $\mathcal{D}_\mu$ and $\mathcal{D}_\nu$ on such a doublet, and prove that this takes the form of an infinitesimal gauge transformation in which the parameter is replaced by $-\vec{F}_{\mu\nu}$.