Examen Stochastische processen
1 februari 2018 NM

Naam:..................

- Schrijf je antwoorden op genummerde pagina's. Schrijf je naam op elke bladzijde en start een nieuwe pagina bij elke vraag. Kladwerk dien je ook in, maar apart.
- Het examen is schriftelijk met open boek (zonder boeken).

1. Consider the Markov diffusion process for a position $x_t \in \mathbb{R}$,

$$\dot{x}_t = -U'(x_t) + 2\,\xi_t$$

where ξ_t is white noise and $U(x) = x^2$.
a) What is the stationary distribution?
b) At time one ($t = 1$) we put $x_1 = 0$. Find the time-correlation function $\langle x_t\, x_s \rangle$ for all $1 \leq t \leq s$.

2. Consider a collection of spins, each having two possible values, $\sigma_i = \pm 1$ for $i = 1, \ldots, N$. Each discrete time we randomly pick a spin from that collection and we flip it with probability $0 < p < 1$. So for example, if at time $n + 1$ we happen to pick the spin with label i we flip it as $\sigma_i(n+1) = -\sigma_i(n)$ with probability p, while all the other spins remain then untouched. Consider then the magnetization

$$M(n) = \sum_{i=1}^{N} \sigma_i(n)$$

as function of time $n = 0, 1, 2, \ldots$. Show that $M(n)$ is a Markov chain. Specify its transition probabilities and find its stationary probability law.

3. At time zero a Poisson process $N(t)$ is started with rate μ; $N(0) = 0$. Suppose that (independently of $N(t)$) $X(t)$ is a two-level Markov process, $X(t) \in \{0, 1\}$, with rates $k(0, 1) = a, k(1, 0) = b$, and started from $X(0) = 1$. What is the probability that $X(t) = 1$ during the whole time-period where $N(t) = 1$?

4. Consider a random walker on a ring with N sites in continuous time. The rate to move one step to the right (clockwise) is $p = \psi(E) \exp E/2$, and the rate to move one step to the left (counter clockwise) is $q = \psi(E) \exp -E/2$. Here, $E \geq 0$ is a parameter (external driving field) and $\psi > 0$ is a positive function of E.
Compute the clockwise stationary current $j(E)$ as a function of E (and also depending possibly on the function ψ).
How should we choose the function ψ so that we get negative differential conductivity for large E, i.e., so that

$$\frac{dj}{dE} < 0$$

for large E. You can give an example that works.

5. Show that for all observables f,

$$L(f^2) \geq 2\, f\, Lf$$

for the generator L of a Markov jump process.