Afternoon exam 28 January 2014, stochastic processes

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1. We consider the Markov chain on $K = \{+1, -1\}$ with transition probability

$$p(x, y) = \frac{1}{Z} e^{2xy}$$

where Z takes care of the normalization and needs to be calculated.

a) What is the numerical value of the probability of a trajectory $(x_0, x_1, \ldots...)$ with $x_0 = x_1 = x_2 = 1, x_4 = x_5 = -1, x_6 = x_8 = 1$ when initially at time zero we draw the value of $x_0 = 1$ with probability $1/3$?

b) Is this chain detailed balance, and what is the stationary distribution?

2. Consider a network with four states (x, v) where $x \in \{0, 1\}, v \in \{-1, +1\}$. (Imagine x to be a position and v like a velocity.) We define a Markov process in continuous time via transition rates that depend on parameter $b > 0$,

$$k((1,+1),(1,-1)) = k((1,-1),(1,+1)) = k((0,+1),(0,-1)) = k((0,-1),(0,+1)) = 1$$

$$k((1,-1),(0,-1)) = k((0,+1),(1,+1)) = b$$

All other transitions are forbidden.

a) Determine the stationary distribution on the four states as function of b.

b) Is there detailed balance?