Life insurance mathematics

01/02/2024

1 Question 1

You are given the select-and-ultimate mortality table, with 2 year select period.

x	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
60	0,09	0,11	0,13	62
61	0,1	0,12	0,14	63
62	0,11	0,13	0,15	64
63	0,12	0,14	0,16	65

1. Calculate ${}_{0,2}q_{[60]+0,7}$ under the assumption of UDD.
2. Calculate ${}_{0,7}q_{[61]+0,7}$ under the assumption of constant force of mortality.
3. Proof $e_x = p_x(1 + e_{x+1})$

2 Question 2

You are given an extract of select life table, with 2 year select period. Assume an interest rate of 6%. Deaths are uniform distributed over each year of age.

x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$	x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
40	99 327.82	99 283.06	99 229.76	42	72	88 846.72	87 852.03	86 627.64	74
41	99 274.69	99 226.72	99 169.41	43	73	87 656.25	86 555.99	85 203.46	75
42	99 217.72	99 166.14	99 104.33	44	74	86 339.55	85 124.37	83 632.89	76
43	99 156.42	99 100.80	99 033.94	45	75	84 885.49	83 545.75	81 904.34	77
44	99 090.27	99 030.10	98 957.57	46	76	83 282.61	81 808.54	80 006.23	78
45	99 018.67	98 953.40	98 874.50	47	77	81 519.30	79 901.17	77 927.35	79
46	98 940.96	98 869.96	98 783.91	48	78	79 584.04	77 812.44	75 657.16	80
47	98 856.38	98 778.94	98 684.88	49	79	77 465.70	75 531.88	73 186.31	81
48	98 764.09	98 679.44	98 576.37	50	80	75 153.97	73 050.22	70 507.19	82

1. Describe the present value of the following insurance benefits in words.

$$Z = \begin{cases} 0 & \text{if } T_x \leqslant 5 \\ 100000v^{T_x} & \text{if } 5 < T_x \leqslant 35 \\ 50000v^{35} & \text{if } T_x > 35 \end{cases}$$

2. Give the expected present value in actuarial notation.

3. Suppose one single premium of 15 000. Show that this premium exceeds the present value of the insurance benefits, provided that

$$T_x \leqslant 5 \qquad\qquad T_x \geqslant 32,56.$$

4. Use the life table and the UDD assumption. Calculate the probability that where this one premium is sufficient to cover the present value of the benefits for a select life of 45.

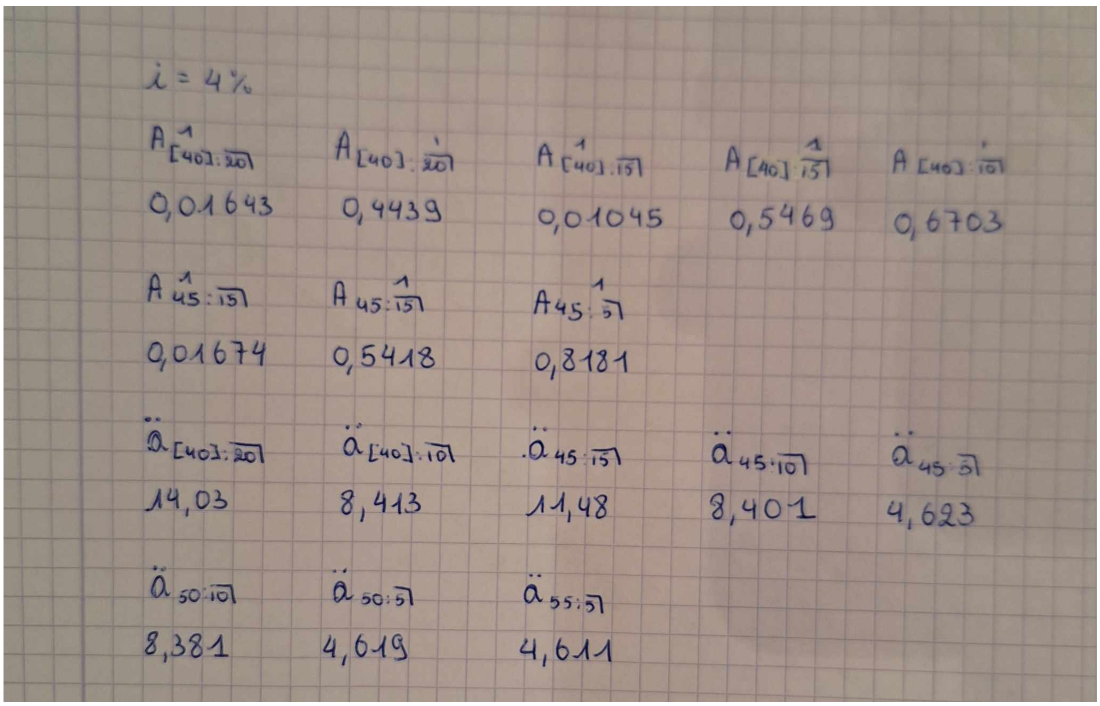
3 Question 3

Consider a 20-year term insurance issued to a select life aged 40, with sum insured 50 000 payable at the end of the year of death. If the policyholder is still alive at age of 60, a sum of 25 000 is insured. Premiums are payable annually in advance. In the first 10 years level premiums equal to P are payable. After 10 years this premium is reduced to $0.5P$ payable for the rest of the term.

We use the following premium basis:

- interest: 4% per year effective
- initial expenses: 20 % of the gross premium plus an extra 100
- renewal expenses: 5% of the gross premiums, the first excluded, plus 25, annually from the second onward
- expenses at benefit payment: 100

You are given the following values, calculated at an interest rate of 4%:



$i = 4\%$

$A^{1}_{[40]:\overline{20}\rvert}$	$A_{[40]:\overline{20}\rvert}^{\ \ \ 1}$	$A^{1}_{[40]:\overline{15}\rvert}$	$A_{[40]:\overline{15}\rvert}^{\ \ \ 1}$	$A_{[40]:\overline{10}\rvert}^{\ \ \ 1}$
0,01643	0,4439	0,01045	0,5469	0,6703
$A^{1}_{45:\overline{15}\rvert}$	$A_{45:\overline{15}\rvert}^{\ \ \ 1}$	$A_{45:\overline{5}\rvert}^{\ \ \ 1}$		
0,01674	0,5418	0,8181		
$\ddot{a}_{[40]:\overline{20}\rvert}$	$\ddot{a}_{[40]:\overline{10}\rvert}$	$\ddot{a}_{45:\overline{15}\rvert}$	$\ddot{a}_{45:\overline{10}\rvert}$	$\ddot{a}_{45:\overline{5}\rvert}$
14,03	8,413	11,48	8,401	4,623
$\ddot{a}_{50:\overline{10}\rvert}$	$\ddot{a}_{50:\overline{5}\rvert}$	$\ddot{a}_{55:\overline{5}\rvert}$		
8,381	4,619	4,611		

1. Calculate the net premium.
2. Show that the gross premium is equal to 1179.
3. Calculate the gross premium policy value at time $t = 5$, using the premium basis.
4. Explain why the policy value at time $t = 5$ increases/decreases if we change the interest rate to 6%.
5. Calculate the asset share at time $t = 1$, using the premium basis. Given is a mortality rate equal to $q_{[40]} = 0,00053605$. What is the policy value at time $t = 1$?