Exam quantum mechanics

June 2016

You can use the book. Be clear, what is not clear cannot be corrected. You can answer in Dutch, French or English. If you want a Dutch translation of the exam questions, feel free to ask. Good luck!

Noethers theorem [10 pts].

Discuss the equivalent of Noethers theorem in quantum mechanics [5pts]. Apply it to conservation of momentum and angular momentum [4 pts]. What about a free particle in a box with periodic boundary conditions [1pts]?

Identical particles [4ptn]

Consider two electrons for which all quantum numbers, but the spin, are known to be equal. What is the resulting spin of the two-electron system. Explain and *calculate* your answer.

Non-degenerate perturbation theory [6 ptn]

Consider the one-dimensional harmonic oscillator with Hamiltonian

$$H = \frac{p_x^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \gamma\frac{m^2\omega^3}{\hbar}x^4 \equiv H_0 + H' \ . \tag{1}$$

a) What is the dimension of γ? [0.5 pnt]
b) Suppose we want to consider H' a perturbation. What is the order parameter of the perturbative expansion, and what is the condition to justify the treatment of H' as a perturbation? [0.5pnt]
c) What are the conditions on n such that the matrix elements:

$$\langle 0|H'|n\rangle \tag{2}$$

are nonzero? [2pnt]
d) Compute the first and second order effect of the perturbation H' on the ground state $|0\rangle$. What is its value, in case $\gamma = 0.1$, in units of $\hbar\omega$? Compare it with the value $E_0 = 0.559$ obtained by numerical integration of the Schrödinger equation for the full Hamiltonian H. [3pnt]
e) We change the perturbation to become cubic: $H'' = \gamma\frac{m^2\omega^3}{\hbar}x^3$. What is the dimension of γ, now? Rearrange the physical constants in order to identify a new combination $\gamma'(\gamma, m, \omega, \hbar)$ which is dimensionless.[1 pnt]